A Reformulation of the Cost Plus Net Value Change *(C+NVC)* Model of Wildfire Economics

Geoffrey H. Donovan and Douglas B. Rideout

ABSTRACT. The Cost plus Net Value Change *(C+NVC)* model provides the theoretical foundation for wildland fire economics and provides the basis for the National Fire Management Analysis System (NFMAS). The *C+NVC* model is based on the earlier least Cost plus Loss model *(LC+L)* expressed by Sparhawk (1925). Mathematical and graphical analysis of the *LC+L* model illustrates two errors in model formulation. First, suppression is incorrectly modeled as a model output. Second, suppression and primary protection are incorrectly modeled as negatively correlated. These errors are shown to be perpetuated by the contemporary *C+NVC* model and to have serious implications for the model's capacity to correctly identify the most efficient level of fire management expenditure. A corrected graphical representation of the *C+NVC* model is presented, which allows the most efficient level of fire management expenditure to be correctly identified. FOR. SCI. 49(2):318–323.

Key Words: Economics, fire, cost plus net value change.

ECONOMIC THEORY has long played an important role in establishing federal wildfire management budgets. This role has increased in significance over the last 30 yr in response to consistently rising wildfire management budget requests. For example, between the periods 1970–1974 and 1991–1995, the Forest Service's average annual expenditure on fire management rose fivefold from $106,574,520 to $537,161,067 (Schuster 1997). The U.S. Senate Appropriations Committee in its review of the 1979 Forest Service fire management budget request noted this trend of increasing expenditures and mandated the Forest Service to conduct cost-benefit analysis on all future budget requests (USDA Forest Service 1995).

The Forest Service responded by developing the National Fire Management Analysis System (NFMAS), a computerized fire budgeting and planning tool (USDA Forest Service 1995). NFMAS was designed to calculate the most efficient annual fire management budget[1] for a given geographical area, by minimizing the sum of all monetized wildfire related costs. The sum of all wildfire related costs is known as "Cost plus Net Value Change" or *C+NVC*, where *C* denotes all costs associated with fire suppression, and NVC[2] denotes net fire related damages. Increasing expenditures on fire suppression *(C)* are intended to reduce net fire related damages *(NVC)*, and the optimal level of suppression and damage is that which minimizes total cost (Simard 1976). The *C+NVC* model is closely related to, and derived from, the earlier Least Cost Plus Loss model *(LC+L)* illustrated by Sparhawk (1925). Since Sparhawk's original work, there have been significant improvements in micro-economic theory, including the introduction of comparative statics by Samuelson (1947). Further, the importance of the *C+NVC* model has increased with time, reflecting its evolution from the theoretical basis for wildfire economics to additionally providing the conceptual foundation for NFMAS.

The increasing magnitude of fire management budgets and significant improvements in micro-economic theory warrant a thorough review of the *C+NVC* and *LC+L* models. This article employs graphical and comparative static techniques to analyze the *C+NVC* and *LC+L* models, and suggests potentially significant improvements. The analysis focuses on the implications of the theoretical structure of the

The authors are research forester, USDA Forest Service, Pacific Northwest Research Station, Portland, OR, and professor, Department of Forest Science, Colorado State University, Fort Collins, CO 80523. Geoffrey H. Donovan can be reached at (503)808-2043; E-mail: gdonovan@fs.fed.us.

C+NVC model and how improvements to this structure can be incorporated into the model to ensure the correct identification of the most efficient level of fire management expenditure.

Model Specification

The following analysis focuses on how the *LC+L* and *C+NVC* models treat decision variables. Therefore, a careful definition of these variables, their relationship to each other, and their effect on the *NVC* function is developed first. Presuppression expenditures are analogous to capital expenditures in the theory of the firm. We define presuppression as representing expenditures on wildfire management prior to the start of a fire season. Purchasing a fire engine and purchasing equipment for a hand crew are examples of presuppression expenditures. Other authors have used a different definition of presuppression, which includes initial attack expenditures. Consequently, suppression expenditures represent expenditures on extended attack. This definition could be used just as well for the following analysis, because presupression and suppression are still independent model inputs related through the *NVC* function. The definition used by the authors was selected because it more closely matches the manner in which capital and labor are treated in the theory of the firm.

In contrast, we define suppression as representing direct fire fighting expenditures during a fire season. Wages for a hand crew and fuel for an airtanker are each examples of suppression expenditures. Suppression expenditures are analogous to labor expenditures in the theory of the firm.

Suppression and presuppression are independent model inputs related through the *NVC* function. Independence of presuppression and suppression means that a particular level of one input does not determine the level of the other. For example, the purchase of a fire engine (presuppression expenditure) prior to a fire season does not determine how many times that engine will be used during the fire season (suppression expenditure). However, the level of presuppression may affect the *optimal* level of suppression through the *NVC* function [Equation (1) assumes that $\partial NVC/\partial P$ and $\partial NVC/\partial S$ are less than zero]:

$$NVC = f(P, S) \tag{1}$$

The relationship between presuppression, suppression, and *NVC* in Equation (1) may be further clarified using a level curve. The level curve in Figure 1 illustrates the relationship between suppression and presuppression for a fixed level of *NVC*.

The rate of substitution between presuppression and suppression is defined as:

$$\frac{-f_s}{f_p} = \frac{dp}{ds} \tag{2}$$

where f_s denotes the marginal net reduction in damage from a marginal increase in suppression and f_p denotes the same for presuppression. Equation (2) defines the slope of the level curve in Figure 1 as being equal to the negative of the ratio of
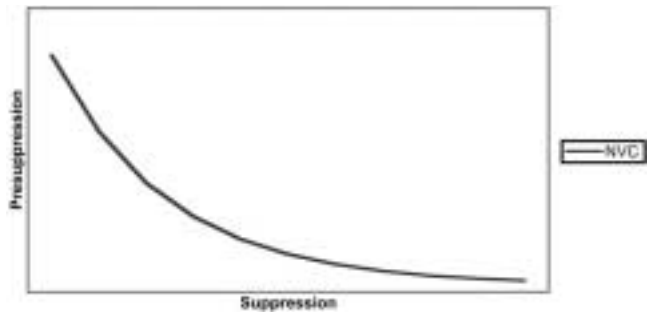

Figure 1. *NVC* level curve.

the marginal contributions of suppression and presuppression in reducing damage. For a constant level of *NVC,* presuppression can be substituted for suppression. It should be carefully noted that this relationship between presuppression and suppression only holds for the special case in which *NVC* is held constant.

Sparhawk uses different terminology to define the variables in his *LC+L* model.

> Protection costs are in two distinct categories. One, which may be called primary protection, includes the cost of the organization for prevention detection, and suppression (including personnel, equipment, and improvements), and is determined in advance. The second includes actual costs of suppression, such as temporary labor, subsistence, and transportation, as well as the time of forest officers taken off from other work.

The third model variable is defined as, "…losses incurred in spite of protection…." These definitions indicate that primary protection, suppression, and loss (*LC+L* model) are equivalent to presuppression, suppression, and *NVC* in the *C+NVC* model (with the exception that the *LC+L* model does not explicitly consider potentially positive effects of wildfire). Sparhawk defines an additional aggregate variable total liability, "[the] sum of suppression costs plus losses."

Carefully defining the relationship between model variables facilitates a more meaningful examination of the *LC+L* and *C+NVC* models.

Sparhawk's *LC+L* Model

The objective of Sparhawk's *LC+L* model (Sparhawk 1925) was to, "determine how much money can justifiably be spent for fire protection on national forests." Using an illustration (Figure 2), he showed that expenditures on a fire protection organization (primary protection *and* suppression) could be justified so long as the sum of these expenditures and loss was declining.

We show that Figure 2 reflects two interrelated misconceptions found in Sparhawk (1925) regarding the relationships between the three model variables (primary protection, suppression, and loss). Each misconception has important implications for the integrity of the *LC+L* and *C+NVC* models. Hence, each is developed below.

The first misconception in Figure 2 is that suppression is expressed as a model output, solely dependent on fire occurrence. This misconception is reflected in his statement that, "These costs [suppression], like losses, can not be determined in advance, but together with the losses depend upon
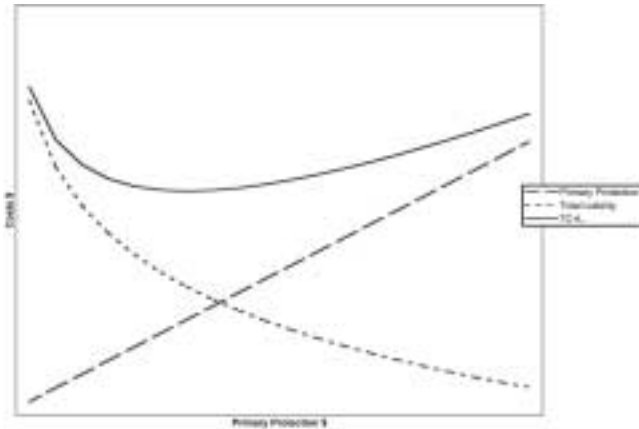
Figure 2. Sparhawk's Least Cost Plus Loss model.

the occurrence of fires." This assertion led Sparhawk to treat suppression as a model output rather than a decision variable. Observing a correlation between suppression and fire occurrence, he inferred a causal relationship. Although fire managers often respond to severe fire seasons with a heavier utilization of suppression resources, this response remains a management decision variable. It is therefore incorrect to model suppression as a model output. Simard (1976) recognized suppression as a model input in creating a total cost function linking costs to fire management effort.

Second, Figure 2 illustrates suppression and primary protection as negatively correlated; i.e., primary protection increases as suppression declines.[3] It seems probable that Sparhawk modeled primary protection and suppression in this manner on a consideration of the different combinations of primary protection and suppression that could result in a given amount of loss. For a fixed amount of loss, a decrease in primary protection (suppression) expenditures could be offset by an increase in suppression (primary protection) expenditures. However, since loss is not held constant in Sparhawk's *LC+L* model, modeling primary protection and suppression as negatively correlated cannot be justified.

The implications of these errors in model formulation can be more precisely specified when the model is expressed by a mathematical objective function. Although Sparhawk did not present an objective function for the minimization of all fire-related costs and losses, one is implied by Figure 2. Examination of the implied objective function in (3) helps to identify the consequences of Sparhawk's errors in model formulation, which are obscured by graphical representations:

$$MIN(C + L) = P(P) + [S + L](P) \quad (3)$$

The terms *S, P, L,* and *C* denote suppression, primary protection, loss, and total protection, respectively. Equation (3) cannot identify the optimal levels of primary protection, suppression, and loss for two reasons. First, to correctly identify the optimal level of suppression it must be multiplied by its factor price. This cannot be done within the framework of Equation (3) because suppression and loss are summed. Changing the factor price of suppression would change the optimal level of suppression employed in the optimal solution. Therefore, as Equation (3) cannot correctly accommo-

date the factor price of suppression it cannot correctly identify optimal levels of suppression, primary protection, and loss. Second, combining suppression and loss into one function means that optimal levels of both variables cannot be identified, since an infinite number of combinations of the two could sum to a particular value of suppression plus loss.

Contemporary *C+NVC* Model

Sparhawk's errors in model formulation have implications for the contemporary *C+NVC* model. As with Sparhawk's *LC+L* model, examination of the implied objective function of the *C+NVC* model helps to identify the implications of Sparhawk's original errors. In addition, a comparison is made between the comparative statics of the *C+NVC* objective function and the comparative statics of an alternative objective function in which presuppression and suppression are treated as independent model inputs. This comparison aids in determining the capacity of the contemporary *C+NVC* model to identify optimal levels of presuppression, suppression, and *NVC*.

The contemporary *C+NVC* model contains two important revisions to the *LC+L* model. First, Sparhawk's total liability function is separated into two functions: a suppression function and an *NVC* function (Figure 3). Second, the *NVC* function denotes the net effects of wildfire, to allow for the inclusion of beneficial impacts (Simard 1976). In addition, primary protection is referred to as presuppression.

Although these revisions are important to contemporary expressions of wildfire economics and planning, they do not correct the errors in Sparhawk's 1925 formulation. However, the revisions result in Sparhawk's errors having different implications in the context of the *C+NVC* model and its implied objective function:

$$MIN(C + NVC) = W^p P + W^s S(P) + NVC(P, S(P)) \quad (4)$$

The dependent relationship between suppression and presuppression in (4) has become accepted as a legitimate approach to the fire management problem. For example, Pyne (1996) in describing the theory of *LC+L* states, "It assumes, in essence, [a] relationship between the investment in fire protection (variously measured) and its returns in the form of suppression costs and resource damages (again
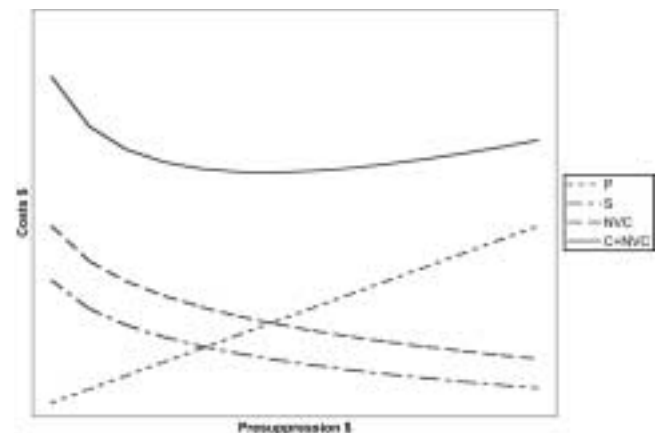

Figure 3. Contemporary *C+NVC* illustration.

variously measured)." To illustrate the implications of such dependence (4) is contrasted with an objective function (5) where S and P are treated as independent inputs related through the NVC function (Rideout and Omi 1990):

$$MIN: C + NVC = W^p P + W^s S + NVC(P, S) \qquad (5)$$

Differentiating (5) with respect to P and then S gives the first order conditions (6) and (7):

$$\frac{\partial(C + NVC)}{\partial P} = W^p + \frac{\partial NVC}{\partial P} = 0 \qquad (6)$$

$$\frac{\partial(C + NVC)}{\partial S} = W^s + \frac{\partial NVC}{\partial S} = 0 \qquad (7)$$

The second order conditions are expressed in (8) and (9):

$$\frac{\partial^2(C + NVC)}{\partial P^2} = \frac{\partial^2 NVC}{\partial P^2} \succ 0 \qquad (8)$$

$$\frac{\partial^2(C + NVC)}{\partial S^2} = \frac{\partial^2 NVC}{\partial S^2} \succ 0 \qquad (9)$$

If the NVC function is convex with respect to P and S, then the $C+NVC$ function will have a minimum.

Identification of the minimum of the $C+NVC$ function in the restricted case (suppression is dependent on presuppression, a condition common to the $LC+L$ and $C+NVC$ models) is represented by (4). Equation (4) can only be differentiated with respect to P:

$$\frac{\partial(C + NVC)}{\partial P} = W^p + W^s \frac{dS}{dP} + \frac{\partial NVC}{\partial P} + \frac{\partial NVC}{\partial S} \frac{dS}{dP}$$
$$= 0 \qquad (10)$$

Differentiating again with respect to P yields the following second order condition:

$$\frac{\partial^2(C + NVC)}{\partial P^2} = \frac{d^2 S}{dP^2}(W^s + \frac{\partial NVC}{\partial S}) + \frac{\partial^2 NVC}{\partial P^2} \succ 0 \qquad (11)$$

Convexity of the NVC function with respect to P and S is sufficient to ensure that (8) and (9) are positive. However, convexity of the NVC function will not ensure that (11) is positive. The presence of the first derivative of the NVC function with respect to suppression ($\partial NVC/\partial S$ which is assumed negative) means that the relative magnitudes of the elements of the second order condition will determine its sign. Should

$$\left(W^s + \frac{\partial NVC}{\partial S}\right) = 0$$

then there would be no ambiguity concerning the sign of (11). However, fulfillment of the first order condition (10) ensures that this will not be the case. Therefore, the dependent relationship between presuppression and suppression [ex-

pressed in (4)] may result in a $C+NVC$ function without a global minimum.

Further, even if the restricted $C+NVC$ function has a minimum, it may not coincide with the minimum of the unrestricted $C+NVC$ function. To illustrate this point, Figure 4 charts an unrestricted and a restricted $C+NVC$ function. Both are drawn using the same NVC function

$$NVC = 50 - \left(4P^{0.4}S^{0.4}\right)$$

but the restricted $C+NVC$ function includes a dependent relationship between suppression and presuppression

$$S = 23 - 7LN(P)$$

(points for both graphs were generated using a linear program. Parameters for the NVC and S functions were chosen to provide a clear illustrative example.).

Figure 4 shows that the restricted function lies on or above the unrestricted function for all values of P, and that the minima of the two functions occur at significantly different values of P (8 restricted, 10.5 unrestricted).

Therefore, the dependent relationship between presuppression and suppression, stemming from Sparhawk's original errors in specifying the $LC+L$ model, has two implications for the $C+NVC$ model. First, the restricted $C+NVC$ function may not have a minimum. Second, even if the restricted $C+NVC$ function does have a minimum, it should not be expected to coincide with the minimum of the unrestricted $C+NVC$ function.

C+NVC Corrected

To correctly illustrate the $C+NVC$ model, presuppression and suppression should be modeled as independent inputs (Rideout and Omi 1990). That is, both should be allowed to vary independently, while remaining related through the NVC function. The envelope approach shown in Figure 5 allows the relationship between the three variables to be correctly illustrated in two dimensions.

Figure 5 shows three $C+NVC$ curves, each drawn with progressively higher, fixed levels of presuppression. The lower envelope of the $C+NVC$ curves drawn in this manner will trace out the unrestricted $C+NVC$ function (P and S independent).
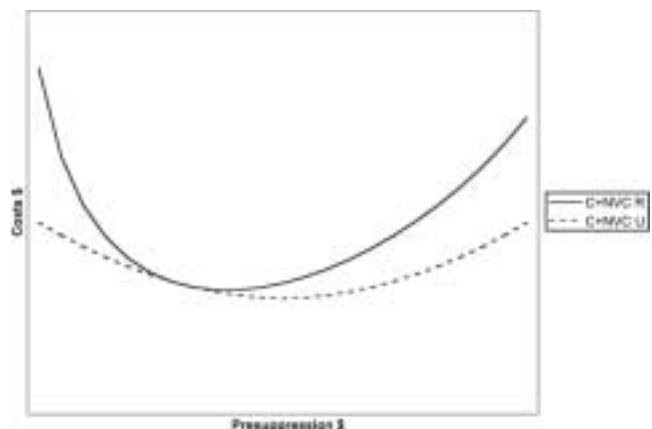

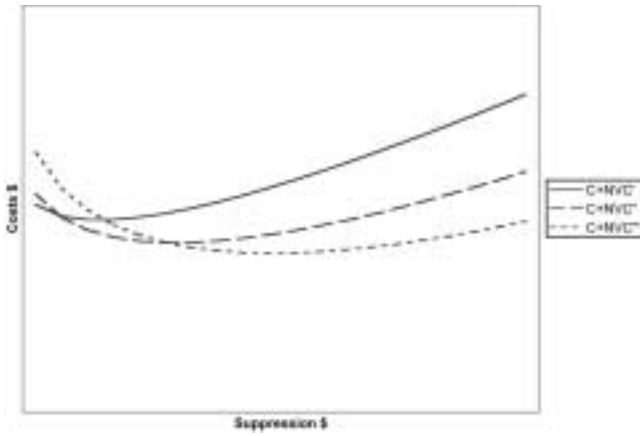**Figure 4. Comparison of restricted and unrestricted C+NVC functions.**

Figure 5. Envelope of *C+NVC* curves with fixed levels of presuppression.

This envelope approach illustrates the relationship between *P, S* and *NVC* in two dimensions. The relationship between the *C+NVC* functions with fixed levels of presuppression and the unrestricted *C+NVC* function they trace out may be expressed mathematically using the envelope theorem.

The functions in Figure 5, plotted with fixed levels of presuppression, may expressed by:

$$C + NVC^f(S, \overline{P}) \qquad (12)$$

The lower envelope of these functions, that traces out the unrestricted *C+NVC* function, may be expressed by:

$$C + NVC(P, S) \qquad (13)$$

The envelope theorem indicates the following relationship between functions (12) and (13) (assuming the level of *S* employed is optimal):

$$\frac{\partial(C + NVC)}{\partial P} = \frac{\partial(C + NVC^f)}{\partial P} \qquad (14)$$

Figure 5 does not illustrate individual *P, S,* and *NVC* functions. To illustrate these functions in two dimensions one of the variables must be held constant. Figure 6 shows the relationship between *S* and *NVC* with *P* fixed at its optimal level.

Comparison of Figure 6 (unrestricted *C+NVC* function) and Figure 3 (contemporary *C+NVC* graphical representa-
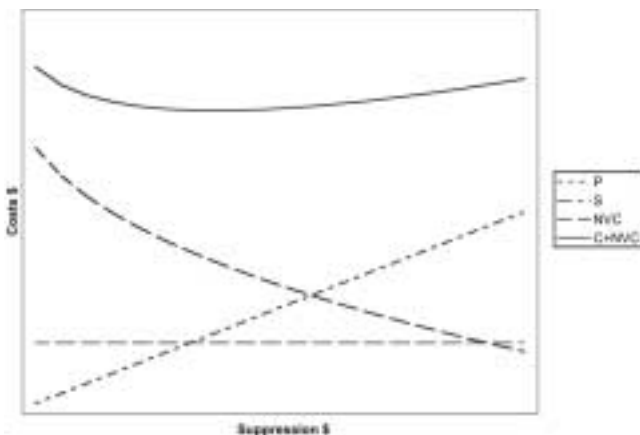

Figure 6. Corrected illustration of *C+NVC* model.

tion) reveals important differences related to Sparhawk's original errors. Figure 3 treats suppression as a model output, while in Figure 6, suppression is treated as a model input. Figure 3 models suppression and presuppression as negatively correlated, while Figure 6 allows only suppression to vary. Consequently Figure 6 identifies the global minimum of the *C+NVC* function, while Figure 3 does not.

Discussion

The purpose of this article is not to unduly criticize Sparhawk's *LC+L* model. The fact that this is being written over 75 yr later speaks to the enduring importance of Sparhawk's model. Rather, it is the purpose here to identify the source of errors in the contemporary *C+NVC* model and to demonstrate the significance of these errors.

Sparhawk's *LC+L* model is shown to have two errors in model formulation. These errors result from misconceptions about the relationships between the variables in the model, and the model's scope. The use of a graphical representation as opposed to a mathematical formulation masked the inability of the *LC+L* model to identify the optimal levels of suppression, primary protection and loss. Failure to identify Sparhawk's original errors has led to their perpetuation by contemporary *C+NVC* expositions.

The *C+NVC* model separates Sparhawk's suppression plus loss function into an *NVC* and a suppression function. As a result, Sparhawk's errors in model formulation have different consequences for the *C+NVC* model. First, the objective function may not have a minimum value, preventing the identification of the optimal levels of presuppression, suppression, and *NVC*. Second, the objective function may have a minimum value, but this may occur at an inefficient level of presuppression, thus misidentifying the optimal levels of presuppression, suppression, and *NVC*.

The *C+NVC* model provides the theoretical foundation for NFMAS, and thus any errors in model formulation could have significant implications for the integrity of the NFMAS process. For example, NFMAS does not allow suppression and presuppression to be modeled independently. The modifications presented here correcting current errors in the *C+NVC* model could improve the NFAMS process, and any future applications of the *C+NVC* model.

The *C+NVC* model is a strategic level model that solves for optimal levels of presuppression, suppression, and *NVC*. The model does not provide information on how these optimal budgets should be allocated (Gonzalez-Caban 1986). Future work could valuably focus on extending the scope of the *C+NVC* model so that it could address this problem. Such a broadening of the model's scope would be helpful in designing operational optimization models for use in wildfire budgeting and planning.

Endnotes
[1] The level of fire management expenditure that minimizes the sum of all fire related costs and damages.
[2] *NVC* is a net cost function, so is defined as costs minus benefits.
[3] If loss were to decline sufficiently in response to increasing levels of primary protection, then suppression and primary protection need not be negatively correlated. However, historically this is how the relationship between primary protection and suppression has been interpreted (see Figure 2).

Literature Cited

GONZALEZ-CABAN, A. 1986. Developing fire management mixes for fire program planning. USDA For. Serv. Gen. Tech. Rep. PSW-88. 8 p.

MILLS, T.J. 1979. P. 75–89 *in* Proceedings, Symp. on fire control in the 80's. Intermountain Fire Council, Missoula, MT.

MILLS, T.J. 1986. Wildfire impacts on the present net value of timber stands: Illustrations in the Northern Rocky Mountains. For. Sci. 32(3):707–724.

PYNE, S.J., P.L. ANDREWS, AND R.D. LAVEN. 1996. Introduction to wildland fire. Wiley, New York. P. 428.

RIDEOUT, D.B., AND P.N. OMI. 1990. Alternate expressions for the economic theory of forest fire management. For. Sci. 36(3): 614–624.

SAMUELSON, P.A. 1947. Foundations of economic analysis. Harvard University Press, Boston. 447 p.

SCHUSTER, E.G., D.A. CLEAVES, AND E.F. BELL. 1997. Analysis of USDA Forest Service fire-related expenditures 1970–1995. Res. Pap. PSW-RP-230. 29 p.

SIMARD, A.J. 1976. Wildland fire management: The economics of policy alternatives. Can. For. Serv. Tech. Rep. 15. Ottawa, Ontario. 52 p.

SPARHAWK, W.N. 1925. The use of liability ratings in planning forest fire protection. J. Agric. Res. 30(8):693–792.

USDA FOREST SERVICE. 1985. National Fire Management analysis systems users' guide of the initial action assessment. (FPL-IAA 2.2). USDA For. Serv., Washington, DC.